Properties of Jointly-Blue Noise Masks and Applications to Color Halftoning

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With the emergence of blue noise halftoning as a preferred halftoning technique, the issues of Moiré patterns in conventional ordered dither color halftoning are replaced by new issues concerning the quality of overlaid blue noise patterns. The goal of blue noise mask design in color halftoning is to generate a set of blue noise masks that produce high quality blue noise patterns whether they are used individually or jointly. We first address this question by inspecting various combinations of unstructured binary patterns, then explore the elementary properties of combinations of blue noise binary patterns. Based on the above analysis, an algorithm using filter techniques is proposed. This algorithm generates a set of jointly blue noise masks (JBNM) and each mask of the JBNM will be applied to one color plane. The JBNM possesses the properties such that each individual mask is blue noise, also the combination of the masks produce color images with blue noise characteristics, e.g., the dots from different color planes are mutually exclusive and maximally dispersed at highlight levels. Examples of halftoned color patches using JBNM and other schemes are illustrated. The evaluation demonstrates that JBNM can produce high quality color halftone patterns with little texture and graininess.

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Introduction

Color halftoning is an extension of gray scale halftoning, however it entails additional complications. Color rendering is the synthesis of halftoning processes in several color planes. Typically, cyan, magenta, yellow and optional black along with "high fidelity" or accent colors are halftoned independently and combined on a single page to produce color hardcopy. The combination of color planes is not a simple superimposition of all the color dots. The appearance of color images relies on the interactions of color planes. For different halftone techniques, the interactions between color planes can produce undesirable impacts on the halftoned output image. For color halftoning using conventional periodic screens, such as clustered dot halftoning, the dither matrix possesses its own periodic structures and inherent frequency, thus the combinations of two or more color planes can result in undesirable artifacts such as Moiré patterns. Moiré patterns are induced by the interference of low frequency components of the periodic patterns and even a slight misregistration of color planes will cause objectionable artifacts.[1,2] Early work on color halftone with periodic dithering techniques focused on eliminating or reducing such adverse artifacts. The solution is to rotate halftone screens so that the screens used for different color planes are separated from each other by some angles. The combination of the color screens oriented at different angles will result in so called "rosette" patterns, which are round shaped patterns with a higher frequency than that of Moiré. Usually, the screen angles are oriented at ±15° for cyan and magenta, 45° for black and 0° for yellow to minimize the visibility of the rosette patterns.[3] For color halftoning with blue noise techniques, the problems caused by the combinations of periodic screens can be avoided because the combinations of stochastic patterns will not produce periodic low frequency contents.

Blue noise halftoning has recently become the widely accepted technique in digital printing technology. Compared to ordered dither halftoning, one of the advantages of blue noise halftoning is that it is capable of producing visually pleasing patterns without adding its own structures to the halftoned images. Various halftone techniques have been developed to generate blue noise patterns, such as error diffusion (Floyd and Steinberg,[4] Knox and Eschbach,[5] Wong,[6]), blue noise mask (BNM) (Mitsa and Parker,[7,8] Yao and Parker,[9]), direct binary search (DBS) (Analoui and Allebach[10]) and void-cluster method (Ulichney[11]), etc. Also, numerous studies have considered the spectral properties of binary patterns and other quality factors in blue noise pattern design. In Ref. 12, Ulichney introduced the terms of "blue noise" and principle frequency in digital halftoning and analyzed the spectral characteristics of well-formed dither patterns. The spectral properties and the edge-enhanced characteristic of error diffusion were explained by Knox.[13] The association of the appearance of binary textures with their power spectra was examined by Dalton.[14] Sullivan, Ray and Miller applied visual cost function to minimize the visual modulation errors of binary patterns.[15] Yu, Parker and Yao,[16] Yu and Parker[17] explored the optimality of filter design in minimizing the visual errors of individual pattern and in acquiring uniformity of the pattern quality over the whole gray scale. These analyses and explorations were

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applied and integrated in the development of the algorithms to generate high quality, visually pleasing blue noise patterns that exhibit specific spectral characteristics and small visual errors.

In the color halftoning with blue noise, the combination of different color planes is, to some extent, a synthesis of blue noise patterns. Compared to the studies of blue noise on gray scale halftoning, the properties of the analysis and synthesis of multiple, superimposed blue noise patterns have received less attention, despite the critical importance. Although the blue noise properties were well described in image and frequency domains, the characteristics of the blue noise patterns are not automatically guaranteed in combined planes. The goal in synthesis is to somehow generate a halftone scheme that will create a visually pleasing pattern for each color plane, and also create a visually pleasing joint pattern when the combination of the color planes is required. Suppose a separate mask is applied for each color plane. We require that the quality of blue noise is preserved in each mask, and the resulting combination patterns also exhibit high quality blue noise appearance, at least in the ideal case. For example, a 10% light gray patch requires printing 10% of all the CMY primaries, whereas a light red patch requires printing 10% of two of the primaries, magenta and yellow, whereas a light cyan requires printing 10% of cyan dots only. For each case, the output halftoned image should be high quality blue noise pattern whenever a 10% gray, red, green, blue, cyan, magenta or yellow is called for in any combination of color overlays. This requirement is more significant for light density regions since clumps or graininess at highlight levels are highly visible.

The contents and organization of this paper are oriented toward a better understanding of the synthesis of combined blue noise patterns and applying the synthesis properties in generating jointly blue noise masks. To provide background, current schemes that have been used in color halftoning will be reviewed. Then examples of simple combinations of stochastic patterns will be given to illustrate the possibilities that arise in combining unstructured binary patterns. Subsequently, a power spectral analysis of combining binary patterns is explored. An approach is then proposed to address the synthesis of jointly-blue noise masks, such that the masks are individually blue noise and also produce blue noise patterns when they are combined in the usual sense of color overlay. Finally, examples of utilizing those schemes and jointly blue noise masks are provided and their quality evaluations are illustrated.

Current Color halftoning schemes with Blue Noise Mask

A number of different schemes have been proposed to generate one or more blue noise matrices for color halftoning (Parker and Mitsa,[19] Yao and Parker,[20] Yu, Parker and Yao,[21] Spaulding, Miller and Schildkraut,[22] Lin and Allebach[23] and Wang and Parker[24]).

Dot-on-Dot

An intuitive and simplest realization of color dithering is the "dot-on-dot" scheme. In this scheme, one single BNM is used for all color channels. This scheme generates the most visible patterns at highlight levels. For example, if a 10% neutral color patch is halftoned, then the dots from cyan, magenta and yellow planes will be printed on the same positions. The overlay of the three-color dots will produce a pattern with 10% black dots on the combined plane, so the resulting pattern is highly visible. Also this method is vulnerable to misregistration of color planes.

Shifted Masks

Spatially shifted versions of masks can be used for each color channel to minimize the number of the dots that are overlaid.[19,20] For example, a blue noise mask can be used for the cyan color channel. Then the mask is shifted by some number of pixels and then applied to the magenta channel. This shift is circularly periodic and can be implemented on either the vertical or horizontal direction or both. A new mask for yellow or other color planes can be acquired by shifting different number of pixels. Care must be taken when choosing shift values so that no inadvertent low frequency artifacts are added in the combinations of color planes.

Inverted Masks

Another approach is to use an "inverted mask" to apply to one or more channels.[19,20] The inverse mask is generated by inverting or taking the 255 complement of a mask (assume a 8 bit mask), i.e.,

$$m_{inv}(i,j) = 255 - m(i,j) \tag{1}$$

where $m(i,j)$ is a BNM and $m_{inv}(i,j)$ is its inverted version.

This inverted mask is also a blue noise mask. Applying a mask and its inverted version to different color planes will reduce the number of dots that are overlaid for two of more color planes. In light gray level, it is guaranteed that the dots from the second color plane will not overlay on the dots from the first color plane and they will typically be placed on the voids of the dots from the first color plane. This non-overlaying distribution causes small visual errors. However, this scheme can be applied to only two color planes. For the other color planes, shifted version masks can be used as the supplementary scheme.

Mutually Exclusive Masks (Four-Masks)

The dot-on-dot, shifted and inverted masks use either a single blue noise mask or a blue noise mask and some transformations of that mask. In four-masks, a set of masks is derived. Complementary random seeds are used as the initial patterns in the blue noise generating process.[20,21] These seed patterns have the properties of being mutually exclusive at some level g. For example, in a 4 color CMYK system with 4 masks, the four seed patterns can be generated by double-thresholding a blue noise mask. The positions of the dots which have the values between [0, 63], [64, 127], [128, 191] and [192, 255] are labeled as the positions of the minority dots of the first pattern, the second pattern, the third pattern and the fourth pattern, respectively. Thus, the four seed patterns have a dot fill density of 25% and they are mutually exclusive from each other. Starting with these four mutually exclusive seed patterns, four blue noise masks are constructed independently. The color dots generated by the four masks will not overlap if the levels of all color planes are less than 25%.

Consequences of Combinations

In many color printing applications, the choice of technique has been largely empirical, but in all cases the rendering results in some overlay or combination of multiple blue noise patterns, like the schemes described in last section, in which the overlaid blue noise patterns can be dependent or independent of each other. This raises the issue of the blue noise "quality" (or lack

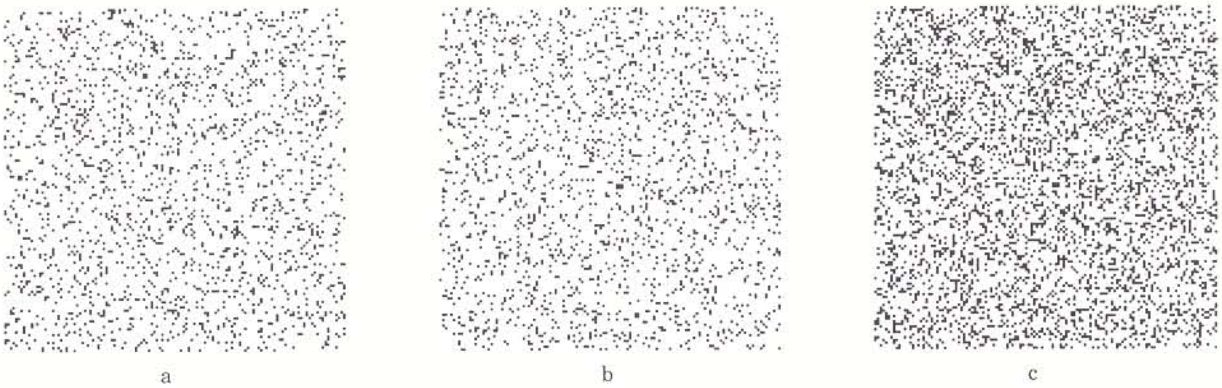


Figure 1. An example where the combination of two white noise patterns results in white noise pattern. (a) white noise No. 1; (b) white noise No. 2; and (c) overlay of the two patterns.

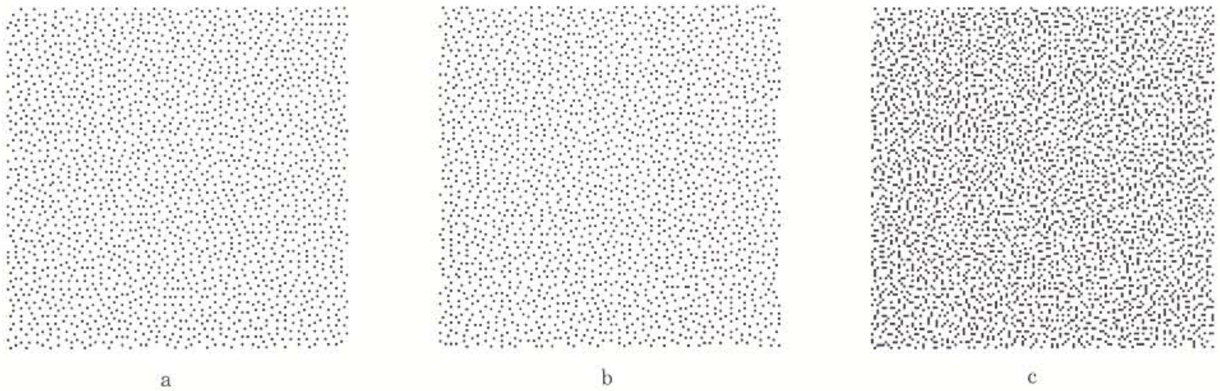


Figure 2. An example where the combination of two blue noise patterns results in a relatively "grainy" or poor quality blue noise. (a) blue noise No. 1; (b) blue noise No. 2; and (c) overlay of the two patterns.

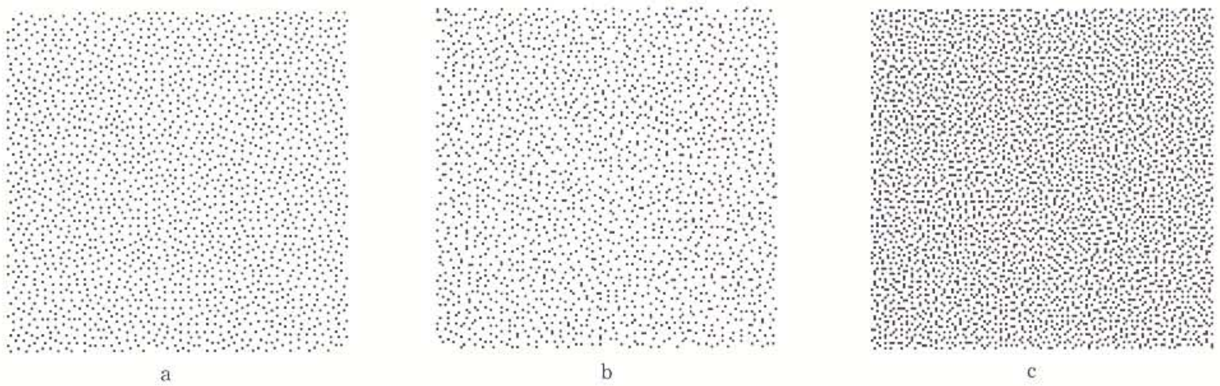


Figure 3. The combination of a good quality blue noise pattern (a), and a relatively "grainy" pattern, (b) can produce a good quality blue noise pattern (c).

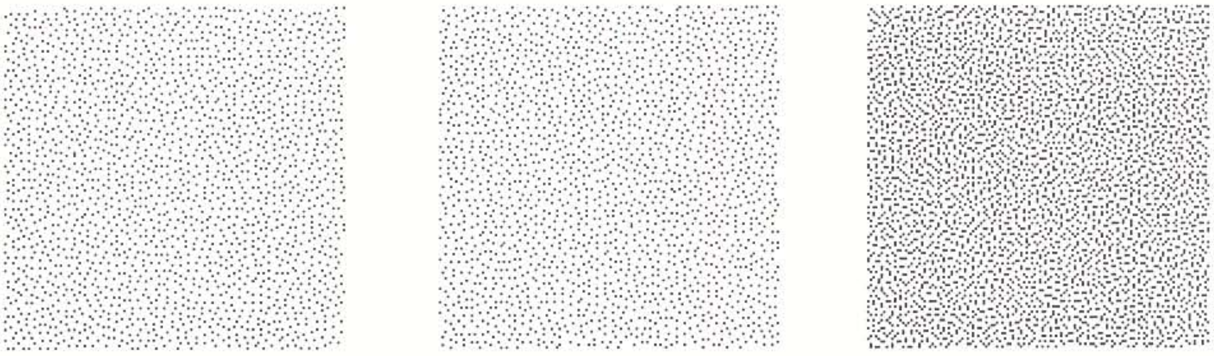
thereof) of the superimposition of multiple blue noise patterns. To address this question, we will consider some visual examples. Underlying mathematical and theoretical concepts of blue noise patterns and their combinations are explored in next section.

We address the properties of two mutually exclusive "daughter" patterns and a "parent" pattern formed by overlaying the two "daughter" patterns. Although this is a simplified situation of the combining procedure in the practical color halftoning process, it can illustrate some basic combination issues.

First, in Fig. 1 we demonstrate that the addition of two white noise patterns produces a white noise pattern.

Perhaps what is less understood is the complexities in combining blue noise patterns. Figure 2 demonstrates that the addition of two blue noise patterns can result in a poor quality binary pattern, that is to say a pattern with excessive "graininess".

Also what is interesting is the fact that the combination of a blue noise pattern plus a relatively "grainy" blue noise pattern can produce a good blue noise result. Figure 3 provides an example of this type of combination. In the figure, the left pattern has good blue noise characteristics, so does the combined pattern on the right, although the middle pattern is not a high quality blue noise pattern.



a b c

Figure 4. The combination of two good blue noise patterns can produce a high quality blue noise pattern. (a) blue noise No. 1, (b) blue noise No. 2 and (c) overlay of the two patterns.

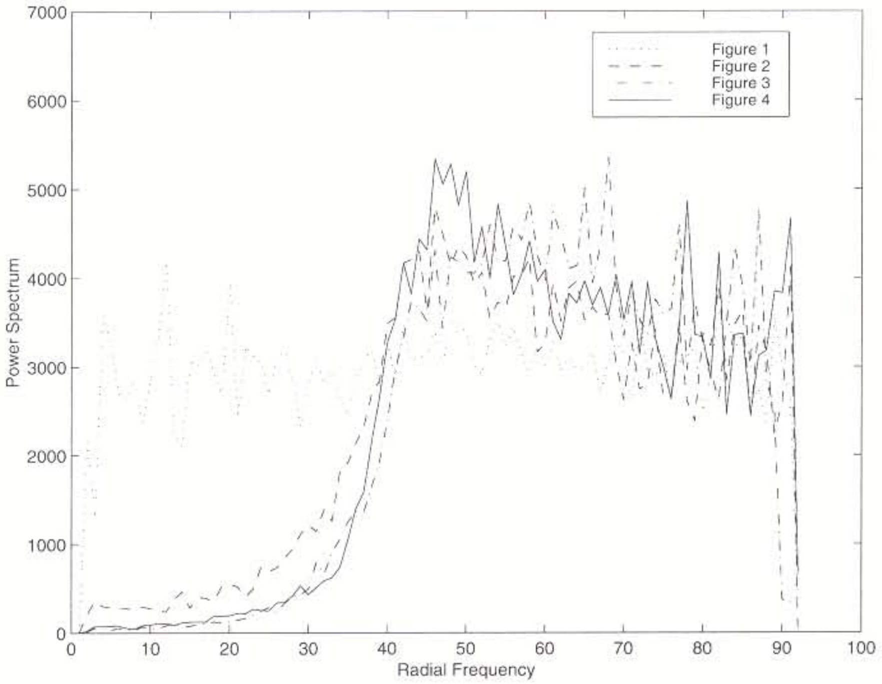


Figure 5. The Radial Average Power Spectra of the patterns in Figs. 1 through 4. Only the RAPS of the combined patterns are plotted.

As a final example, two blue noise patterns, if properly constructed, can result in a blue noise combination, as shown in Fig. 4.

In designing blue noise masks for color applications, our task is to build a set of patterns, such that both of the daughter patterns and parent patterns are high quality blue noise, like the patterns illustrated in Fig. 4. The term of "blue noise quality" we refer to so far is not precisely defined yet. A high blue noise quality should be a visually pleasing pattern, without texture, clumps and other artifacts in image domain.

Table I lists the human visual system (HVS) model based frequency weighted mean square error (FWMSE). An HVS based FWMSE gives a quantitative measurement of the error between the frequency weighted halftoned pattern and the original continuous image. The HVS simulates the human visual perception as a linear filter. The value of printer resolution is chosen as 300 dpi and that of the viewing distance is chosen as 15 in.. The formula of HVS model[15] used in the calculation is:

TABLE I. MSE by HVS Model

	MSE of d_1 ($\times 10^{-3}$)	MSE of d_2 ($\times 10^{-3}$)	MSE of combined pattern ($\times 10^{-3}$)
Figure 1	15.916	15.330	26.217
Figure 2	4.597	5.112	9.164
Figure 3	4.469	6.198	5.670
Figure 4	4.852	4.836	6.343

$$H(i,j) = \begin{cases} 2.2(0.192 + 0.114f)\exp[-(0.114f)^{1.1}] & \text{if } f > f_{\max} \\ 1.0 & \text{otherwise} \end{cases} \quad (2)$$

where the unit of f is cycle/degree.

The radial average power spectra (RAPS) of the four examples are given in Fig. 5. Only those of the parent patterns are displayed. The definition of RAPS is given in the next section. The different characteristics of the RAPS and their corresponding patterns are pronounced

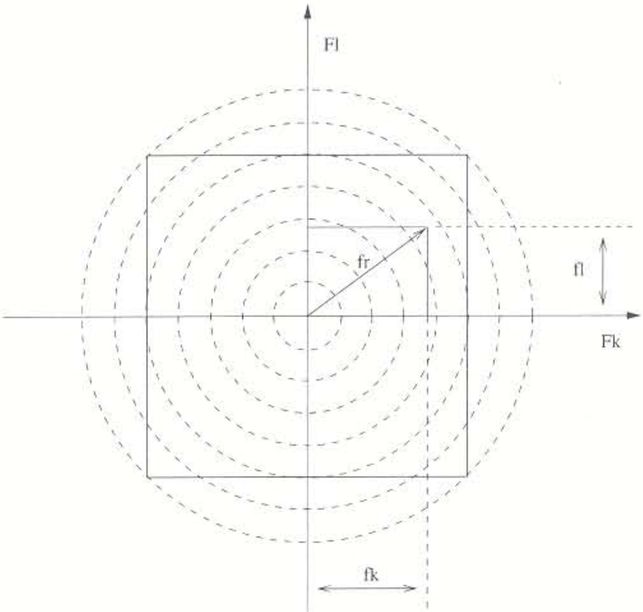


Figure 6. Illustration of the radial frequency.

in both frequency domain and image domain. These examples demonstrate that it is possible to produce a high quality blue noise parent pattern with two blue noise daughter patterns, but it is not guaranteed.

Theoretical Analysis of the Power Spectrum of the Combined Pattern

The stochastic halftone pattern can be modeled as a stationary random process. The power spectrum of the stochastic halftone pattern is defined as the Fourier transform of the autocorrelation of the halftone pattern. We use $R(k,l)$ to represent the power spectrum. The notation k and l denote the horizontal and vertical directions in the two dimensional frequency space. A one-dimensional representation of the power spectrum, namely the radial average power spectrum (RAPS), is widely used to characterize the spectral properties of halftone patterns. To analyze the radial power spectrum characteristics, the Fourier space is partitioned into co-centered annuli with the same width, as illustrated in Fig. 6. The central radial frequency of an annulus is f_r and is calculated by:

$$f_r = \sqrt{(f_k)^2 + (f_l)^2} \tag{3}$$

where f_k and f_l are the frequencies in horizontal and vertical directions, respectively. Denote the number of the frequency samples in an annulus as $N_r(f_r)$. Then the radial average power spectrum is the total power spectrum in this annulus divided by the number of samples[12]:

$$R_r(f_r) = \frac{1}{N_r(f_r)} \sum_{\mathrm{int}[\sqrt{f_k{}^2 + f_l{}^2}] = f_r} R(k,l) \tag{4}$$

An important concept in the spectral analysis is the principle frequency.[12] In image domain, the principal frequency f_g corresponds to the average distance between minority pixels under a homogeneous distribution, i.e., it relates to the minority pixel density on image plane. The principal frequency f_g varies from gray level g and has the form of:

$$f_g = \begin{cases} \sqrt{g} & \text{if } g \le 1/2 \\ \sqrt{1-g} & \text{if } g > 1/2 \end{cases} \tag{5}$$

In this equation, g is normalized from 0 to 1. The feature of blue noise is that most energy is concentrated in high frequencies, and the energy in low frequencies is negligible.[12] There exists an energy transition band from a cut-off frequency f_c, where the energy starts to be non-negligible, to principal frequency f_g. Too long or too sharp a transition band will generally cause either graininess or visible low frequency artifacts, so there is a tradeoff in choosing f_c. Parker, Mitsa and Ulichney[18] demonstrated for one set of experiments that when the cut-off frequency f_c equals f_g scaled by a factor of $1/\sqrt{2}$, i.e., $f_c = 1/\sqrt{2} f_g$, the corresponding binary pattern exhibits little grainy texture and low frequency artifacts.

The RAPS can be thus labeled as three parts: low frequency band, transition band and high frequency band. Figure 7 is a typical RAPS curve of a high quality blue noise pattern. It possesses the following characteristics:
(1) Very little energy in $[0, f_c]$;
(2) A smooth transition from f_c to f_g;
(3) Smooth power spectrum in high frequencies, with a peak distribution at f_g.

A better understanding of blue noise spectral characteristics can be acquired by inspecting the RAPS in Fig. 5, where each of them corresponds to a parent pattern in Figs. 1 through 4, and comparing them with the characteristics described above. The RAPS of the white noise parent pattern in Fig. 1, which is indicated by dotted line, has a considerable amount of energy over the whole frequency range; the RAPS of the grainy parent pattern in Fig. 2, which is indicated by dashed line, has extra energy at the low frequency band, which is responsible for the unpleasing graininess; the RAPS of the two other patterns (the two parent patterns in Fig. 3 and 4), which are indicated by dashed-dotted line and solid line, have a shape which is very close to the ideal RAPS of a blue noise pattern and possess the above three characteristics as shown in Fig. 7. The observations of these RAPS illustrate that, no matter how the particular dot arrangement and distribution of a binary pattern appear to be, a visually pleasing blue noise pattern will generally have specific power spectral characteristics that are related to its gray level.

We have studied the blue noise features in the frequency domain, illustrating the power spectrum characteristics of high quality blue noise patterns. Based on the above analysis, we will explore the frequency characteristics of the combinations of blue noise patterns. The possible results of the synthesis of unstructured patterns were illustrated previously. Among the several possible combinations, our desired type of synthesis is to create a high quality combination from high quality daughter patterns. This raises some fundamental questions: What are the factors governing the quality of the combinations of multiple blue noise patterns? If two blue noise patterns are overlaid, is there any guarantee that the resulting pattern is also high quality blue noise?

Assume patterns $d_1(m,n)$ and $d_2(m,n)$ are two binary blue noise patterns that represent gray patches of the same level g_0. Both the $d_1(m,n)$ and $d_2(m,n)$ exhibit the blue noise characteristics in the frequency domain, and their discrete Fourier transforms (DFT) are denoted as

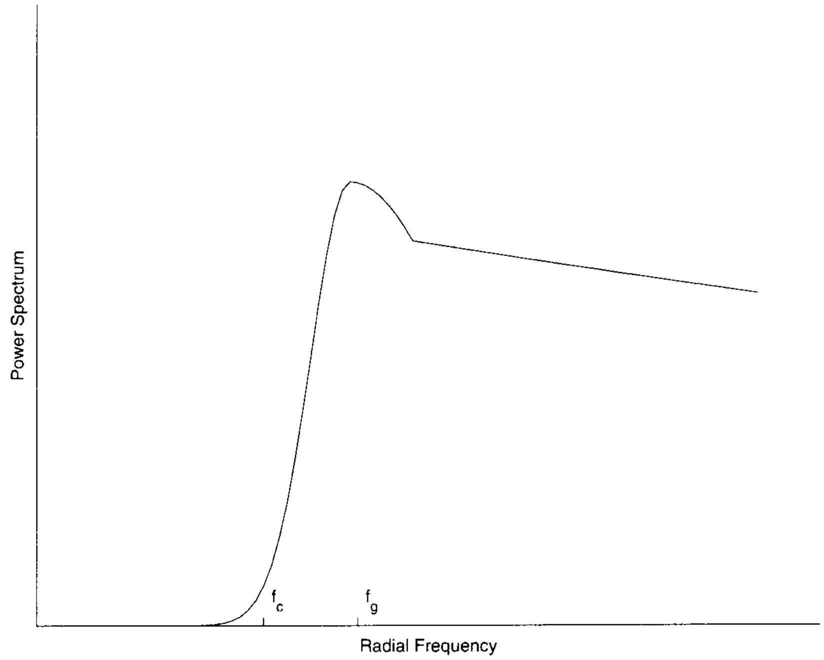


Figure 7. Radial average power spectrum of a typical blue noise pattern.

$D_1(k,l)$ and $D_2(k,l)$, respectively. Overlaying or superimposing these two patterns yields a new pattern $p(m,n)$ which is a rendition of a darker gray level patch of level $g_1 = (1 - 2(1 - g_0)) < g_0$ (assume that we take the white dots as background and overlay the black dots, and that the initial patterns are mutually exclusive in minority pixel locations). Pattern $p(m,n)$'s DFT is $P(k,l)$. Assume the mask size is N by N. The power spectrum of the new pattern $p(m,n)$, $|P(k,l)|^2$, can be derived as:

$$p(m,n) = d_1(m,n) + d_2(m,n) \tag{6}$$

$$P(k,l) = D_1(k,l) + D_2(k,l) \tag{7}$$

where,

$$D_1(k,l) = \sum_{m=0}^{N-1}\sum_{n=0}^{N-1} d_1(m,n)\exp[-j\frac{2\pi mk}{N}]\exp[-j\frac{2\pi nl}{N}] \tag{8}$$

and

$$D_2(k,l) = \sum_{m=0}^{N-1}\sum_{n=0}^{N-1} d_2(m,n)\exp[-j\frac{2\pi mk}{N}]\exp[-j\frac{2\pi nl}{N}] \tag{9}$$

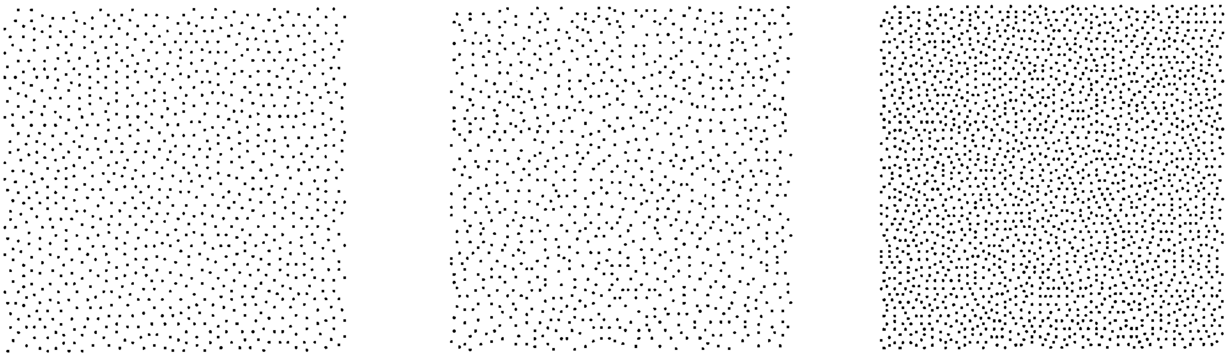
Thus,

$$\begin{aligned} |P(k,l)|^2 &= P(k,l) * P^*(k,l) \\ &= [D_1(k,l) + D_2(k,l)] * [D_1^*(k,l) + D_2^*(k,l)] \\ &= |D_1(k,l)|^2 + |D_2(k,l)|^2 + 2\,\mathrm{Re}[D_1(k,l) * D_2^*(k,l)] \end{aligned} \tag{10}$$

The relationship between the two single patterns and the combination of the two patterns show that the power spectrum of $p(m,n)$ depends not only on the power spectra of $d_1(m,n)$ and $d_2(m,n)$ but also on the Fourier transform of the cross-correlation function between $d_1(m,n)$ and $d_2(m,n)$. The energy induced by the cross-correlation terms can be a major contribution to the power spectrum of superimposed pattern and thus significantly influence the behavior of the superimposed pattern.[25] Although the power spectrum characteristics of the two blue noise patterns $d_1(m,n)$ and $d_2(m,n)$ can be easily assessed in most cases, the behavior of the cross terms between $D_1(k,l)$ and $D_2(k,l)$ is not easily determined from the inspection of individual patterns. When more binary patterns are overlaid together, the problem tends to be more complicated.

The combination of two binary patterns increases the dot density and creates a binary pattern with different gray level, so the principle frequency and the ideal RAPS changes accordingly. The f_g of the parent pattern is larger than that of the daughter patterns, thus it is required the RAPS of the parent pattern "shift" toward high frequency. Then the energy of daughter patterns in the transition band becomes low frequency energy of the parent pattern, so that part of energy will degrade the quality of the parent pattern. To acquire a desired blue noise parent pattern, the cross-correlation terms must compensate or counteract the energy in the frequency range which exists in the daughter patterns but is undesirable for the parent pattern.

In order to illustrate the role of the correlation of daughter patterns in the synthesis of the parent pattern, the RAPS of the set of parent-daughter patterns shown in Fig. 8 are plotted in Fig. 9. As a comparison to the RAPS of the parent pattern, the summation of the RAPS of the two daughter patterns is also plotted, as the dotted line. The gray level of the daughter patterns is 240 (out of 256) and that of the parent pattern is 224. The frequencies are calculated using Eq. 5 and multiplied by the size of DFT which is 128 in this case. Thus, f_g and f_c of the daughter patterns are 32 and 22.6,



a b c

Figure 8. Another example, where two blue noise patterns [(a) and (b)] are overlaid to produce a good quality pattern (c).

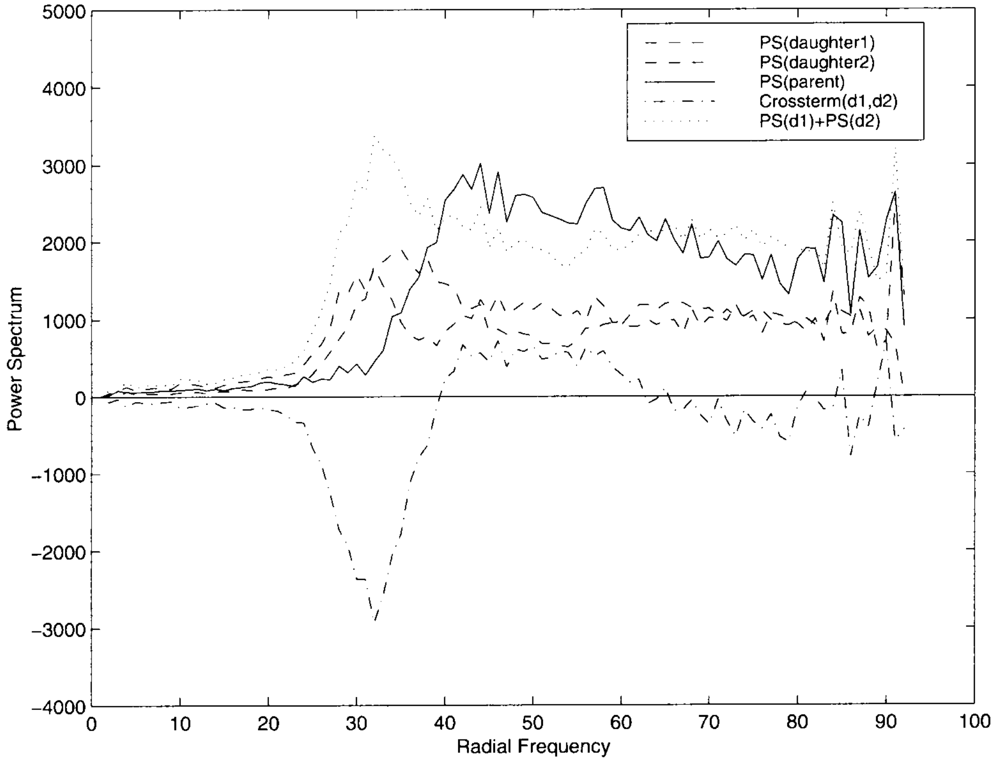


Figure 9. RAPS of parent and two daughter patterns in Fig. 8. and the cross power spectrum between the two daughter patterns. Notice that the negative energy of the cross term cancels the redundant energy around the middle frequency band.

respectively, while those of the parent pattern are 45.3 and 32, respectively. It can be observed that the two daughter patterns have transition energy from 22.6 to 32, while the parent pattern has transition energy from 32 to 46. Their distributions comply with the characteristic curves of their associated gray levels. In the sense of the combined pattern, the energy which exists in the daughter patterns between the cut-off frequencies of the daughter patterns and combined pattern, 22.6 and 32, is not desired for the darker gray level. In this example, the anti-correlation of the daughter patterns, which the dashed-dotted line represents, eliminates most of the energy around this frequency band, along with the energy in the very low frequency band.

This example shows that it is not sufficient to simply combine two well-formed blue noise patterns; to guarantee a well-formed result, the two combined patterns must have a specific anticorrelation. In the image domain, this anticorrelation corresponds to a certain alignment and "interweaving" of minority pixels in the daughter patterns. In the past, filtering operations have been used to ensure anticorrelations (or minority pixel "interweaving") as a single BNM have been constructed.[7–9,17] We will extend the filtering concepts to jointly-blue masks design in the next section.

Constructing Jointly-Blue Noise Masks

The algorithm proposed in this section is based on digital filter techniques of constructing blue noise mask[9] and can be used to generate a set of blue noise masks. The set of blue noise masks is designed jointly to provide high quality color halftone outputs with minimal visibility to human eyes. In grayscale halftoning, one blue noise mask is created by successively adding or removing dots from a single blue noise pattern, whereas in color halftoning, three or four dithering masks are required for most of the color halftoning applications. Like the process of generating a single blue noise mask, the masks are constructed simultaneously by successively adding or removing dots from multiple binary patterns.

The steps to generate jointly optimized blue noise masks are outlined as follows. For the sake of illustration, assume three masks are needed for three-color output channels and a make-down procedure (by adding black dots) is performed.

1. Create three initial masks, m_1, m_2 and m_3, based on three mutually exclusive blue noise binary patterns of same level g. At this level, the minority dots are black dots.
2. Specify an integer M, which is the number of black and white dots to be swapped at each iteration.
3. Construct three binary blue noise patterns s_1, s_2 and s_3 by thresholding the three masks with the level g, and mark the locations of black dots. Change K white dots to black dots for each mask. K is the number of pixels to be changed in order to decrease the gray level by one. For example, if the size of the mask is N by N and the total number of levels is L, then $K=N*N/L$. The K pixels on each mask can be selected randomly as long as they are mutually exclusive on the combined plane at some given levels. This restriction is similar to that described later in step 7.
4. Create the combined patterns. There are total seven combinations for a three mask set: three single patterns s_1, s_2 and s_3, three patterns by combining two single patterns, D_1, D_2 and D_3, and one triple pattern T which is the combination of all the three single patterns. The combined patterns are defined by:

$$\begin{aligned} D_1 &= w_{11} * s_1 \oplus w_{12} * s_2 \\ D_2 &= w_{22} * s_2 \oplus w_{23} * s_3 \\ D_3 &= w_{31} * s_1 \oplus w_{33} * s_3 \end{aligned} \quad (11)$$

$$T = w_1 * s_1 \oplus w_2 * s_2 \oplus w_3 * s_3 \quad (12)$$

The symbol "≈" is the superimposition or overlay operation of minority dots. The factors $w_{11\text{-}33}$ and $w_{1,2,3}$ are weighting factors of the contribution from each single binary pattern. The weighting factors can be chosen according to the requirements of the application. For example, if s_1 corresponds to cyan color plane in the halftoning, w_{11}, w_{31} and w_1 can be chosen proportional to its relative contribution to luminance. The weighting factors provide us a flexibility to fulfill some specific requirements in the mask design.

5. Specify three 2-D low-pass filters L_s, L_d and L_t that are appropriate for single, double and triple patterns, respectively. The dot densities of single and combined patterns should determine the cut-off frequencies of the filters. Apply the filters to the corresponding patterns to obtain seven filtered continuous patterns.
6. Construct three error arrays:

$$E_1 = L_s(s_1) + L_d(D_1 + D_3) + L_t(T) \quad (13)$$

$$E_2 = L_s(s_2) + L_d(D_1 + D_2) + L_t(T) \quad (14)$$

$$E_3 = L_s(s_3) + L_d(D_2 + D_3) + L_t(T) \quad (15)$$

E_1 reflects the synthetic effects on single, double and triple patterns by changing pattern s_1. Likewise, E_2 associates with s_2 and E_3 associates with s_3.

7. Update the three binary patterns sequentially, that means, in one iterative operation, the three patterns are updated one by one. First, update the first pattern, s_1. Sort error array E_1. Find M black dots which have the largest error values and are not marked in step 3, and swap these M black dots with M white dots which have the smallest error values. Similarly, sort error arrays E_2 and E_3 and update s_2 and s_3 accordingly.

   In the above operations, a restriction applies when making the joint masks. For a level higher than a predetermined value, which corresponds to a highlight value, if a pixel is selected by one of the patterns as a black dot, that pixel is not allowed to be selected by the other two patterns any more. The other two patterns have to seek for appropriate candidates other than the pixels that have already been picked. This restriction ensures that the binary patterns of the three masks are mutually exclusive, and the filter technique controls the minority dots to disperse maximally. If the level is decreased to the level that some minority pixels must overlap, then this restriction will not apply.
8. Compute the summation of the MSE of the seven patterns. If the MSE drops, go back to step 4 and repeat the same process. If the MSE increases but $M \neq 1$, set $M = M/2$, and go back to step 4, otherwise, reset M to the initial value as specified in step 2, and go to the next step.
9. Update the three masks based on the three-updated pattern s_1, s_2 and s_3, decrease level by 1, and go back to step 3, until the desired level is reached.

The flowchart of this algorithm is illustrated in Fig. 10. This algorithm creates a set of blue noise masks and the correlations between the masks are controlled to tend to eliminate the redundant energy for the combined level. Either the single or combined pattern exhibits the blue noise characteristics with a desired energy distribution. This is realized by specifying appropriate low-pass filters to single and combined patterns and thus to control the energy shift between single and combined patterns. From the analysis in the previous section, the RAPS of a visually appealing binary pattern typically complies some specific energy distribution. Its cut-off frequency and principle frequency depend on the density of the minority dots on the image plane. In the filter technique to generate blue noise mask, patterns with different levels are assigned different low-pass filters. We call the set of jointly constructed masks "jointly-blue noise masks (JBNM)". The properties of JBNM are that it can produce high quality blue noise patterns whether it is used individually or jointly.

Figure 11 provides an example of the patterns created by 3-jointly-blue noise masks. As a comparison, blue noise patterns of the same levels generated by a single mask are also illustrated. Table II lists the HVS based FWMSE of those patterns. The quality of the jointly blue noise patterns is close to that of the regular blue noise patterns. Table II also demonstrates that the FWMSE's of jointly synthesized patterns are slightly higher than

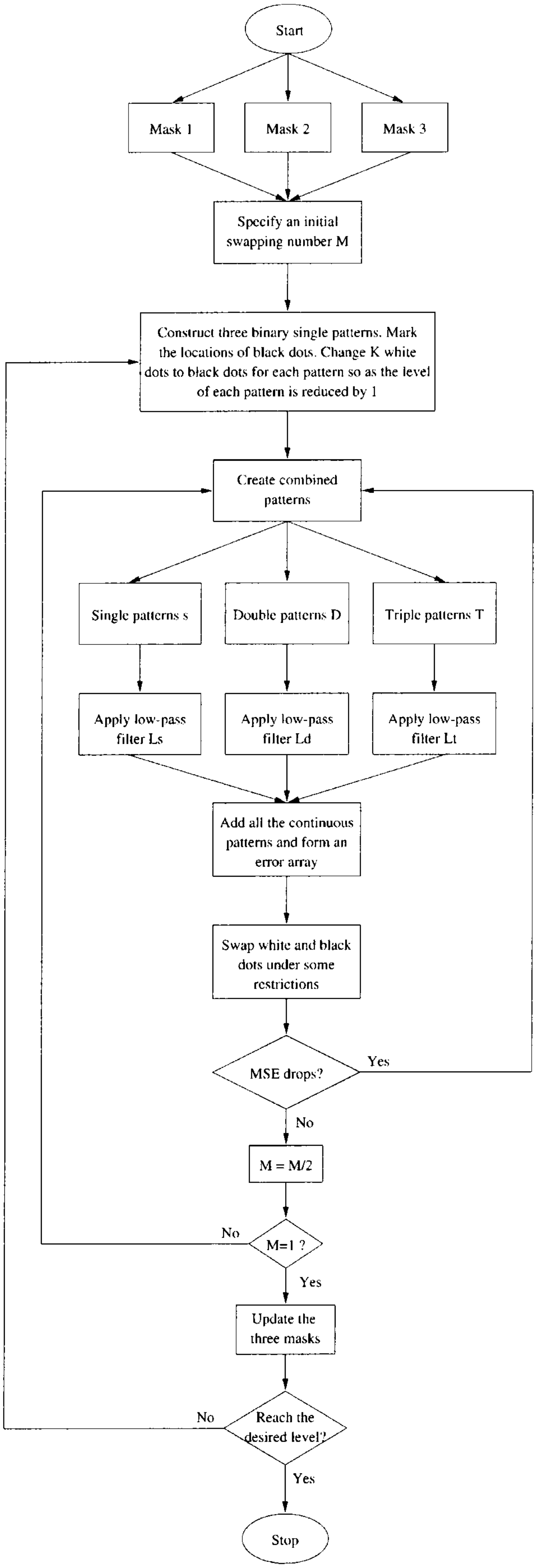


Figure 10. Flowchart of constructing jointly blue noise masks algorithm.

TABLE II. MSE by HVS Model

pattern	MSE × 10^{-3}	pattern	MSE × 10^{-3}	pattern	MSE × 10^{-3}
A	5.042	(A+B)	7.339	(A+B+C)	7.072
B	5.008	(B+C)	7.470		
C	5.027	(C+A)	7.359		
single BN	4.567	single BN	5.670	single BN	7.261

those of regular single patterns. We postulate that this is related to the fact that there are more constraints on available choices in the dot swapping operations while jointly synthesized masks are constructed. In constructing the JBNM, the summation of the MSE of all the single patterns and possible combinations are calculated as the criterion. If swapping one pair of black-white dots reduces the MSE of one of the patterns but increases the overall MSE, such a swap operation will not be performed. There exists a trade-off between the quality of all the patterns and the quality of a particular pattern in joint mask design.

Results of Halftoning a Color Patch

In the previous section, we have proposed an approach to generate JBNM. In this section, we provide examples of applying JBNM and the schemes described in our section reviewing current color halftoning schemes. **Color Plate 15 (p. 386)** shows the results of applying different schemes to render a neutral color solid patch with gray level of 246 (out of 256). The schemes, shifted masks, inverted plus shifted masks, mutually exclusive masks (four-masks) and JBNM are chosen in this example. By inspection, the JBNM results in a smoothest pattern, with very few clumps and little texture and all the dots dispersed maximally on the image plane, while other schemes produce considerable amount of color overlays and color clumps.

The perceived luminance and chrominance errors of these halftoned images are listed in Table III. HVS model was applied on the halftoned images and the luminance and chrominance errors compared with the continuous image were evaluated in CIE$L^*a^*b^*$ space. In the calculation, it is assumed that the resolution is 300 dpi and the viewing distance is 10 in. The JBNM results in the minimal luminance error and its chrominance error is a little higher than those of other schemes.

In the color halftoning schemes discussed earlier, either a single or multiple blue noise masks are applied to several color planes. These schemes use different strategies to produce multiple blue noise masks. The inverted masks and shifted masks are also blue noise. Four-masks are derived from four mutually exclusive blue noise patterns, so the color patterns they generate are mutually exclusive on the combined plane at high levels, and thus the halftoned pattern will cause less visual errors. Although the idea of applying different versions of masks or mutually exclusively derived masks implies the correlation issue discussed in the section of the power spectrum analysis, no specific designs or controls on the correlation between color planes are applied in the construction of multiple masks, or the specific designs are applied only to a few levels but not all the levels, so the optimality of these schemes is not guaranteed.

The JBNM starts from jointly blue noise seed patterns. In the procedure of constructing multiple masks, appropriate low-pass filters are assigned and applied to the single and combined planes to produce high quality jointly blue noise patterns. The jointly optimizing pro-

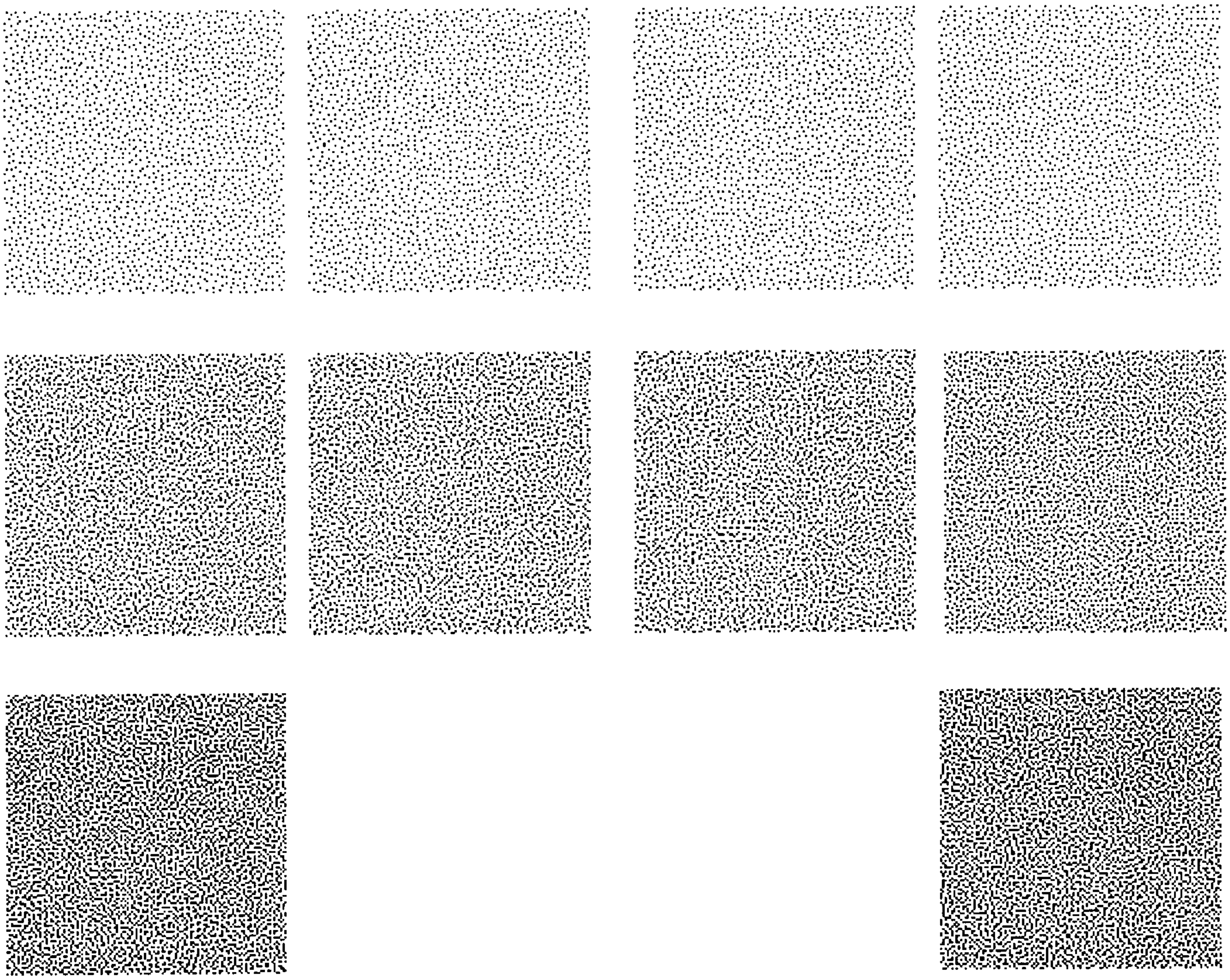


Figure 11. Patterns created by the 3 jointly-blue noise masks A, B and C, and patterns created by a regular blue noise mask. The first row: patterns of $g = 224$. From left to right, patterns created by masks A, B and C, and the pattern created by the single blue noise mask, respectively. The second row: patterns of $g = 192$. From left to right, patterns of combining A and B, combining B and C, combining C and A, and the pattern created by the single blue noise mask, respectively. The third row: patterns of $g = 160$. Left: the pattern of combining A, B and C. Right: the pattern created by the single blue noise mask.

TABLE III. Luminance and chrominance errors of the color images in *Color Plate 22 (p. 386).*

Scheme	ΔL	ΔC
shifted	3.3936	11.7600
inverted + shifted	3.3573	11.8331
four-masks	3.3513	11.8997
JBNM	3.1390	12.5731

cedure is applied to the masks for all the levels, so the JBNM is optimal for not only some specific levels but also for a wide range of levels on both the single and combined planes.

Conclusion

In this article, the current schemes which apply blue noise masks to color halftoning are reviewed.

Various examples are given to illustrate the possibilities of the results of combining binary patterns. Combining different blue noise color patterns can produce both high quality and poor quality combined patterns.

A theoretical analysis explores the relationship between parent pattern and daughter patterns and the spectral properties in the combined plane. The key point of generating a high quality jointly-blue noise pattern is that the correlations between the daughter patterns counteract the energy around a lower frequency band that is inherent from the daughter patterns, but is undesirable for the combined level, regardless the individual power spectrum distribution of daughter patterns.

The algorithm proposed provides a way to control the correlation between the daughter patterns in the procedure of constructing blue noise mask. The jointly blue noise masks are generated simultaneously, and they are optimized for both the single patterns and combined patterns. Thus they produce not only a visually pleasing pattern for a single color plane, but also a visually pleasing joint pattern for multiple color planes.

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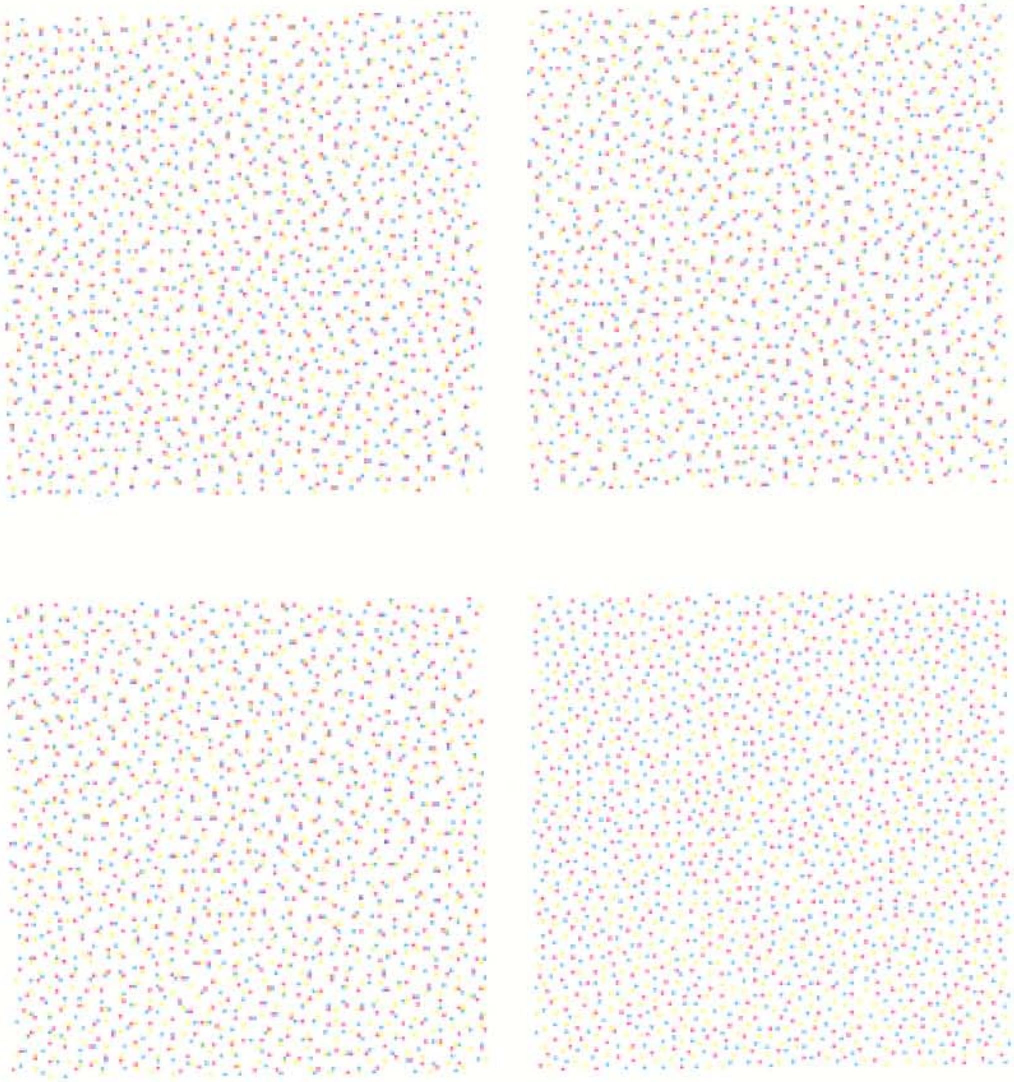

Plate 15. A color patch halftoned with different schemes. From left to right, top to bottom: shifted masks, inverted plus shifted masks, four-masks and JBNM, respectively (Wang and Parker, pp.360–370).